

THT WORKSHEETS

Learn With the Solutions

ATAR Mathematics

Functions, Financial Modelling & Mathematical Reasoning

These solutions explain the mathematical process, the reason each method is appropriate and the meaning of each result in context.

1.

Understanding the Question

We need to solve:

$$4x - 9 = 27$$

Solving an equation means finding the value of x that makes both sides equal.

Our goal is to isolate x .

Step 1: Remove -9

Add 9 to both sides:

$$4x - 9 + 9 = 27 + 9$$

$$4x = 36$$

Step 2: Remove the coefficient 4

Divide both sides by 4:

$$4x \div 4 = 36 \div 4$$

$$x = 9$$

Check

Substitute $x = 9$ into the original equation:

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$$4(9) - 9$$

$$= 36 - 9$$

$$= 27$$

Both sides are equal.

Final Answer: $x = 9$

Common Mistake

When moving -9 to the other side, the operation must be reversed. Add 9; do not subtract another 9.

2.

Understanding the Question

The function is:

$$f(x) = 3x + 5$$

To find a function value, substitute the given input wherever x appears.

a) Find $f(4)$

Substitute $x = 4$:

$$f(4) = 3(4) + 5$$

Multiply first:

$$3 \times 4 = 12$$

Then add 5:

$$12 + 5 = 17$$

Final Answer: $f(4) = 17$

b) Find $f(-2)$

Substitute $x = -2$:

$$f(-2) = 3(-2) + 5$$

Multiply:

$$3 \times -2 = -6$$

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Then add 5:

$$-6 + 5 = -1$$

Final Answer: $f(-2) = -1$

Common Mistake

Use brackets when substituting a negative value:

$$3(-2)$$

This helps prevent sign errors.

3.

Understanding the Question

The cost model is:

$$C = 150 + 25h$$

where:

- C is the total cost;
- h is the number of hiring hours.

A linear cost model usually contains:

- a fixed charge;
- a variable charge that depends on usage.

a) Interpret 150

The number 150 does not contain h, so it is paid regardless of the number of hours.

It represents the fixed hiring fee.

Final Answer: \$150 is the fixed hiring charge.

b) Cost for 8 hours

Substitute $h = 8$:

$$C = 150 + 25(8)$$

Calculate the hourly component:

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$$25 \times 8 = 200$$

Now add the fixed fee:

$$C = 150 + 200$$

$$C = 350$$

Final Answer: \$350

Key Idea

In the model:

$$C = 150 + 25h$$

the gradient, 25, represents the cost added for each extra hour.

4.

Understanding the Question

The investment earns compound interest annually.

Compound interest means that each year, interest is calculated on the growing balance, not only on the original investment.

Principal = \$18,000

Interest rate = 5.2% per annum

Time = 5 years

Convert the rate to a decimal:

$$5.2\% = 0.052$$

Formula

$$A = P(1 + r)^n$$

where:

- A is the final amount;
- P is the principal;
- r is the annual interest rate as a decimal;
- n is the number of compounding periods.

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Substitute

$$A = 18,000(1 + 0.052)^5$$

$$A = 18,000(1.052)^5$$

Calculate:

$$A \approx 23,193.27$$

Final Answer: \$23,193.27

Check

The investment grows for five years at just over 5% annually. A final balance moderately above \$23,000 is reasonable.

Common Mistake

Do not use:

$$18,000 \times 0.052 \times 5$$

That is a simple-interest calculation and does not account for compounding.

5.

Understanding the Question

The line passes through:

(2, 7)

and

(8, 19)

We need to find:

- a) the gradient;
- b) the equation of the line.

a) Find the gradient

Use:

$$m = \text{change in } y \div \text{change in } x$$

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$$m = (19 - 7) \div (8 - 2)$$

$$m = 12 \div 6$$

$$m = 2$$

Final Answer: Gradient = 2

b) Find the equation

Use the linear form:

$$y = mx + c$$

We know $m = 2$, so:

$$y = 2x + c$$

Substitute one known point, (2, 7):

$$7 = 2(2) + c$$

$$7 = 4 + c$$

Subtract 4:

$$c = 3$$

Therefore:

$$y = 2x + 3$$

Check using the second point

When $x = 8$:

$$y = 2(8) + 3$$

$$= 16 + 3$$

$$= 19$$

The equation passes through both points.

Final Answer: $y = 2x + 3$

ATAR Tip

Always verify a line equation using the second given point. This quickly detects gradient or intercept errors.

6.

Understanding the Question

Profit is not the same as revenue.

Use:

$$\text{Profit} = \text{Income} - \text{Total costs}$$

The business sells 120 products.

Selling price per product = \$35

Variable cost per product = \$18

Fixed costs = \$1,450

Step 1: Calculate income

Income:

$$120 \times 35$$

$$= 4,200$$

Step 2: Calculate variable costs

Variable costs:

$$120 \times 18$$

$$= 2,160$$

Step 3: Calculate total costs

Total costs:

$$2,160 + 1,450$$

$$= 3,610$$

Step 4: Calculate profit

Profit:

$$4,200 - 3,610$$

$$= 590$$

Final Answer: \$590 weekly profit

Common Mistake

Do not subtract only the variable costs. Fixed costs must also be included.

7.

Understanding the Question

We need to factorise:

$$x^2 - 11x + 24$$

Then use the factorised form to solve:

$$x^2 - 11x + 24 = 0$$

We need two numbers that:

- multiply to +24;
- add to -11.

The numbers are -3 and -8 because:

$$(-3)(-8) = 24$$

and:

$$-3 + -8 = -11$$

Factorise

$$x^2 - 11x + 24$$

$$= (x - 3)(x - 8)$$

Solve

Set each factor equal to zero:

$$x - 3 = 0$$

$$x = 3$$

or:

$$x - 8 = 0$$

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$$x = 8$$

Final Answer:

$$(x - 3)(x - 8)$$

$$x = 3 \text{ or } x = 8$$

Common Mistake

Because the constant term is positive and the middle term is negative, both signs in the brackets must be negative.

8.

Understanding the Question

The data compares advertising expenditure with weekly sales:

- \$500 advertising → \$4,050 sales
- \$700 advertising → \$4,760 sales
- \$900 advertising → \$5,510 sales
- \$1,100 advertising → \$6,180 sales

We must:

1. describe the relationship;
2. explain why the trend may not continue indefinitely.

Describe the relationship

As advertising expenditure increases, weekly sales also increase.

This is a positive relationship.

The data suggests that greater advertising expenditure is associated with higher weekly sales.

Explain the limitation

This trend may not continue forever because the market may become saturated.

Once most potential customers are already aware of the product, additional advertising may produce only small increases in sales.

Other valid reasons may include:

- limited customer demand;

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- stronger competitors;
- advertising becoming repetitive or ineffective;
- production or stock limitations.

Final Answer: There is a positive relationship between advertising and sales, but the increase may slow because the market can eventually become saturated.

ATAR Tip

A relationship in data does not prove that one variable alone caused the change. Other factors may also influence sales.

9.

Understanding the Question

The loan uses simple interest.

Principal = \$12,000

Rate = 6% per annum

Time = 4 years

Convert the rate:

6% = 0.06

Formula

$$I = P \times r \times t$$

a) Calculate the interest

$$I = 12,000 \times 0.06 \times 4$$

One year of interest:

$$12,000 \times 0.06 = 720$$

Four years:

$$720 \times 4 = 2,880$$

Final Answer: \$2,880 interest

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b) Total amount repaid

Total repayment:

Principal + Interest

$$= 12,000 + 2,880$$

$$= 14,880$$

Final Answer: \$14,880

Common Mistake

The answer to part a is only the interest. Part b requires the original loan amount to be added back.

10.

Understanding the Question

The monthly profit is:

$$P(x) = 450x - 12,000$$

The company makes a profit only when:

$$P(x) > 0$$

We need the smallest whole-number value of x that produces a positive result.

Find the break-even point

Set profit equal to zero:

$$450x - 12,000 = 0$$

Add 12,000:

$$450x = 12,000$$

Divide by 450:

$$x = 26.666\ldots$$

This is the break-even threshold.

The company cannot sell part of a product.

At 26 products, profit is still negative.

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At 27 products:

$$P(27) = 450(27) - 12,000$$

$$= 12,150 - 12,000$$

$$= 150$$

The profit is positive.

Final Answer: 27 products

Key Idea

Do not round 26.67 down. The question asks for the minimum whole number needed to make a profit, so the value must be rounded up.

11.

Understanding the Question

The student claims:

“If two lines have different gradients, they must intersect.”

Parallel lines have the same gradient.

If two distinct lines have different gradients, they cannot be parallel.

In a two-dimensional Cartesian plane, two non-parallel straight lines intersect at exactly one point.

Example

Consider:

$$y = 2x + 1$$

and:

$$y = -x + 4$$

Their gradients are 2 and -1 .

Because the gradients differ, the lines eventually cross.

Final Answer: The statement is always true for two distinct straight lines in the same plane.

Precision Note

The conclusion applies to straight lines in two-dimensional coordinate geometry.

12.

Understanding the Question

The family uses 90 GB of data each month.

The plans are:

Plan A

\$78 per month with unlimited data.

Plan B

\$55 per month plus \$4 for every 10 GB used.

We must calculate the actual monthly cost of each plan.

Plan A

Monthly cost:

\$78

Plan B

Usage:

90 GB

Number of 10 GB units:

$$90 \div 10 = 9$$

Usage charge:

$$9 \times \$4 = \$36$$

Add the base fee:

$$55 + 36 = 91$$

Plan B costs:

\$91 per month

Compare

Plan A = \$78

Plan B = \$91

Difference:

$$91 - 78 = 13$$

Final Answer: Plan A is cheaper by \$13 per month.

Common Mistake

Do not compare only the base fees. Plan B's usage charges must be included.

13.

Understanding the Question

The grade distribution is:

- Grade A: 18 students
- Grade B: 26 students
- Grade C: 31 students
- Grade D: 15 students

We need the percentage of students who achieved Grade B or higher.

“Grade B or higher” includes:

- Grade A;
- Grade B.

Step 1: Find the total number of students

$$18 + 26 + 31 + 15$$

$$= 44 + 46$$

$$= 90$$

Step 2: Find students with B or higher

$$18 + 26 = 44$$

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Step 3: Calculate the percentage

Percentage:

$$44 \div 90 \times 100$$

$$= 48.888\ldots$$

Rounded to one decimal place:

$$48.9\%$$

Final Answer: 48.9%

Common Mistake

Do not include Grade C. “B or higher” means only Grades A and B.

14.

Understanding the Question

The line has:

- gradient -3 ;
- passes through $(4, 5)$.

We need its equation.

Use:

$$y = mx + c$$

The gradient is:

$$m = -3$$

So:

$$y = -3x + c$$

Substitute the point $(4, 5)$:

$$5 = -3(4) + c$$

$$5 = -12 + c$$

Add 12:

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$$c = 17$$

Therefore:

$$y = -3x + 17$$

Check

When $x = 4$:

$$y = -3(4) + 17$$

$$= -12 + 17$$

$$= 5$$

The point lies on the line.

Final Answer: $y = -3x + 17$

ATAR Tip

A negative gradient means the line decreases as x increases.

15.

Understanding the Question

The invoice is \$8,400.

The business can choose:

Option A: receive a 5% discount for immediate payment.

Option B: pay after 12 months with 3% simple interest.

We must compare the total amount paid under each option.

Option A: Immediate payment

Discount:

$$8,400 \times 0.05$$

$$= 420$$

Amount payable:

$$8,400 - 420$$

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$$= 7,980$$

Option A total: \$7,980

Option B: Pay after 12 months

Time = 1 year

Use simple interest:

$$I = P \times r \times t$$

$$I = 8,400 \times 0.03 \times 1$$

$$I = 252$$

Total payment:

$$8,400 + 252$$

$$= 8,652$$

Option B total: \$8,652

Compare the options

Difference:

$$8,652 - 7,980$$

$$= 672$$

Option A costs \$672 less.

Final Answer: Option A is financially better because it saves \$672.

Key Idea

Option A creates a saving, while Option B adds an extra cost. Comparing the final dollar amounts gives a stronger justification than comparing 5% and 3% alone.

Quick Answer Check

1. $x = 9$

2. a) 17

b) -1

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3. a) Fixed hiring fee
b) \$350
4. \$23,193.27
5. a) Gradient = 2
b) $y = 2x + 3$
6. \$590 profit
7. $(x - 3)(x - 8)$; $x = 3$ or 8
8. Positive relationship; market saturation is one valid limitation
9. a) \$2,880 interest
b) \$14,880 total repayment
10. 27 products
11. Always true for two distinct straight lines in the same plane
12. Plan A; cheaper by \$13 per month
13. 48.9%
14. $y = -3x + 17$
15. Option A; saves \$672